

# Disturbance Rejection with Highly Oscillating Second-Order-Like Process, Part X: Cascade Controller with PID and PI sub-controllers

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*Abstract— This is the tenth research paper in a research work investigating the use of specific controllers for possible and efficient disturbance rejection associated with a highly oscillating second-order-like process. This paper presents a series cascade controller utilizing PID and PI sub-controllers. The PID/PI cascade controller is tuned using MATLAB control and optimization toolboxes with five error-based objective functions: ITAE, ISE, IAE, ITSE and ISTSE. The investigated controller has five gain parameters to be tuned. The effect of each controller parameter of the performance of the control system during disturbance rejection is evaluated. The tuned controller is capable of reducing the time response of the closed-loop control system to a unit step disturbance input to 0.00084 (maximum value) at about one second. The PID/PI cascade controller is superior in rejecting disturbance associated with the highly oscillating second-order-like process compared with PD-PI, PI-PD, 2DOF, PPI and PIP controllers.*

*Keywords— Disturbance rejection, highly oscillating second-order-like process, PID/PI series cascade controller, controller tuning, control system performance.*

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## I. INTRODUCTION

Highly oscillating second-order-like process have large maximum percentage overshoot. They represent a challenge for the control engineer since any selected controller has to generate a closed-loop control system having acceptable performance. The purpose of this series of research papers is to investigate using specific controllers to deal with disturbance rejection associated with such highly oscillating processes.

Lee and Park (1998) proposed a method for PID controller tuning based on process models for cascade control systems. They compared with existing methods such as frequency response and ITAE methods [1]. Ellis and Lorenz (1999) explored alternative methods for using some modified forms of cascaded loop controller for use with high performance AC and DC industrial servo-drives. This covered PI-velocity loop cascaded with position loop and PI+velocity controller [2]. Cooper, Rice and Arbogast (2004) compared two architectures of cascade control and feedforward with feedback trim. They presented a comparative example using a jacketed reactor simulation [3]. Johnson and Moradi (2005) studied the concepts of cascade control and its structure. They presented the closed loop transfer function of the cascade control system in terms of all the variables encountered in the system block diagram [4]. Kaya, Tan and Atherton (2007) suggested an improved cascade control structure and controller design based on standard forms to improve the performance of cascade control. They demonstrated the superiority of their method over other two methods [5].

Preitl and Bauer (2007) presented two cascade speed control solutions including a PI-PI cascade control. They used the modulus optimum tuning method to ensure high performance of the control system [6]. Nemati and Bagheri (2010) produced a tuning method for PI controllers in 2DOF structure. Their aim was to have good set-point response, disturbance rejection and maximum robustness to model uncertainties [7]. Cvejn (2011) presented the PI/PID controller for FOPDT plants based on the modulus optimum criterion. He proposed a compensation of the disturbance lag that preserves the stability margin for the disturbance rejection problem [8]. Bhavina, Jamliya and Vashishtha (2013) used a master PI controller for speed control as an outer loop and P controller for current control as an inner loop in cascade structure for a DC motor control [9]. Legweel et. al. (2014) investigated using PIP-cascade controller in HVAC systems and compared with traditional PI and PID controllers. The PIP cascade control resulted in faster response and better performance [10].

## II. PROCESS

The equivalent second-order model to a highly oscillating process step time response has the transfer function,  $G_p(s)$ :

$$G_p(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \quad (1)$$

Where:  $\omega_n$  = process natural frequency rad/s  
 $\zeta$  = process damping ratio

The following process parameters are used as they generate a time response of about 85.4 % maximum overshoot [11]:

$$\omega_n = 10 \text{ rad/s}$$

$$\zeta = 0.05$$

## III. PID/PI CASCADE CONTROLLER

A series cascade controller structure with two disturbance variables, one reference input and one output variable is shown Fig.1 [12].

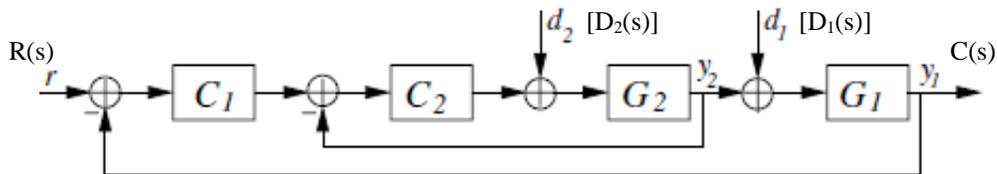


Fig.1 Series cascade controller structure [12].

There are two sub-controllers of the series cascade controller [13]:

*Sub-controller 1:* Is a primary controller of transfer function  $C_1$  {or  $G_{c1}(s)$ }. It is located within a main control loop of the overall block diagram of the control system.

*Sub-controller 2:* Is a secondary controller of transfer function  $C_2$  {or  $G_{c2}(s)$ }. It is located within an internal control loop of the overall block diagram of the control system.

In a previous research work the PI controller design for the primary sub-controller and the P controller design for the secondary sub-controller was selected and the cascade controller was investigated [14]. In another research paper, the author investigated using PI sub-controller as a primary controller and PID sub-controller as a secondary controller [13]. In the present paper the author investigates using the PID sub-controller as the primary controller and the PI sub-controller as the secondary controller. The sub-controllers transfer functions are:

$$G_{c1}(s) = K_{pc1} + K_{i1}/s + K_{d1}s \quad (2)$$

And  $G_{c2}(s) = K_{pc2} + K_{i2}/s \quad (3)$

Where:  $K_{pc1}$  = proportional gain of the primary sub-controller.  
 $K_{i1}$  = integral gain of the primary sub-controller.  
 $K_{d1}$  = derivative gain of the primary sub-controller.  
 $K_{pc2}$  = proportional gain of the secondary sub-controller.  
 $K_{i2}$  = integral gain of the secondary sub-controller.

## IV. CLOSED-LOOP TRANSFER FUNCTION

The dynamics of the closed-loop control system depends on its transfer function between its input and output. For sake of disturbance rejection study, the reference input  $R(s)$  is set to zero in Fig.1. It is assumed that the process has one disturbance variable  $D_1(s)$  and the other process transfer function  $G(s)$  has a unit value and its disturbance variable  $D_2(s)$  is zero. Following those assumptions and using Eqs.1,2 and 3, the closed-loop transfer function  $C(s)/D_1(s)$  is given by:

$$C(s)/D_1(s) = (b_0s^2 + b_1s) / (a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4) \quad (4)$$

Where:  $b_0 = \omega_n^2 (1 + K_{pc2})$   
 $b_1 = \omega_n^2 K_{i2}$   
 $a_0 = 1 + K_{pc2}$   
 $a_1 = K_{i2} + 2\zeta\omega_n(1 + K_{pc2}) + \omega_n^2 K_{d1}K_{pc2}$

$$a_2 = 2\zeta\omega_n K_{i2} + \omega_n^2(1 + K_{pc2}) + \omega_n^2(K_{pc1}K_{pc2} + K_{d1}K_{i2})$$

$$a_3 = \omega_n^2 K_{i2} + \omega_n^2(K_{d1}K_{pc2} + K_{pc1}K_{i2})$$

$$a_4 = \omega_n^2 K_{d1}K_{i2}$$

V. PID/PI CASCADE CONTROLLER TUNING

The PID/PI cascade controller is tuned and the performance of the closed loop control system is evaluated as follows:

- (1) The MATLAB control toolbox is used to evaluate the time response of the closed loop control system to a unit disturbance input using the command ‘step’ [15].
- (2) An objective function is defined in terms of the error between the time response c(t) to a unit disturbance input and a zero value steady-state response. The use of five objective functions is investigated: ITAE, ISE, IAE, ITSE and ISTSE [16] to [18].
- (3) The optimization toolbox is used to minimize each objective function through its command ‘fminunc’ [19]. A sample of the tuning process results is given in Table I for the tuned five controller parameters, maximum time response  $c_{max}$ , time of maximum time response  $T_{cmax}$  and settling time  $T_s$  based on the violation of the 0.05 limit. The values in Table I correspond to a local minimum since the optimization problem is extremely nonlinear and it is expected to have too many local minima. This will be clarified in the rest of the paper through studying the effect of the controller parameters on the performance of the control system incorporating the PID/PI series cascade controller and the highly oscillating second-order process.

TABLE I: PID/PI CASCADE CONTROLLER TUNING

	ITAE	ISE	IAE	ITSE	ISTSE
$K_{pc1}$	9.4308	14.9259	16.5295	9.7697	9.6789
$K_{i1}$	3	3	3	3	3
$K_{d1}$	4.8250	0.0416	1.0426	4.9794	4.9392
$K_{pc2}$	9.9947	10.0001	10.0080	9.9783	9.9889
$K_{i2}$	4.9726	5.0505	4.9811	5.0192	4.9867
$c_{mac}$	0.0772	0.0614	0.0603	0.0747	0.0754
$T_{cmax}$ (s)	0.9240	5.8919	0.4434	0.9198	0.9225
$T_s$ (s)	2.2	40.0	6.0	2.1	2.1

- (4) The effect of the objective function type on the time response of the control system to a unit step disturbance input is shown in Fig.2.

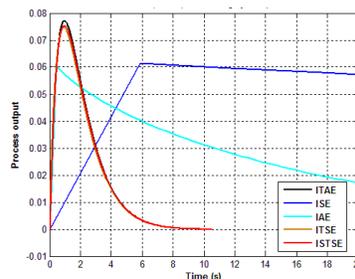


Fig.2 Objective function effect on the control system time response.

- (5) To investigate the effect of the first gain parameter of the PID/PI cascade controller  $K_{pc1}$  on the control system performance, the rest of the optimal parameters are left as in Table I for the ITAE objective function. Then the value of the  $K_{pc1}$  gain is changed in the range:  $100 \leq K_{pc1} \leq 900$ . Sample of the effect of  $K_{pc1}$  on the time response of the control system is shown in Fig.3.

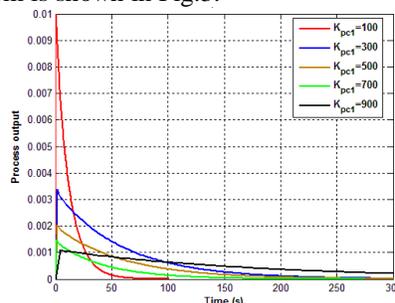


Fig.3 Effect of  $K_{pc1}$  on the disturbance time response of the control system.

- (6) The integral parameter  $K_{i1}$  has no effect on the performance of the control system in terms of its unit disturbance time response.
- (7) The effect of the third parameter  $K_{d1}$  of the controller is investigated for the range:  $109.06 \leq K_{d1} \leq 702.87$  for the first gain parameter set at a value of 800. This effect is shown graphically in Fig.4 for  $K_{d1} = 109.06, 300.67, 503.32$  and  $702.87$ .

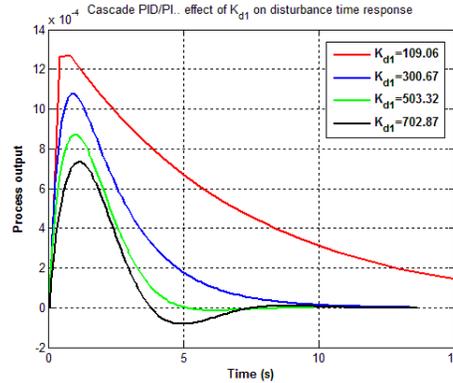


Fig.4 Effect of  $K_{d1}$  on the disturbance time response of the control system.

- (8) The other two parameters  $K_{pc2}$  and  $K_{i2}$  belong to the secondary PI sub-controller of the PID/PI series cascade controller. The effect of the fourth parameter  $K_{pc2}$  of the PID/PI controller of the performance of the control system is shown in Fig.5 for  $K_{pc2}$  in the range:  $2.60 \leq K_{pc2} \leq 52.83$  for the first gain parameter set at a value of 800 and the derivative gain  $K_{d1}$  set at 503. This effect is shown graphically in Fig.5 for  $K_{pc2} = 2.60, 4.09, 30.12, 38.54$  and  $52.83$ . This effect is limited compared with that of the first and third gain parameters  $K_{pc1}$  and  $K_{d1}$ .

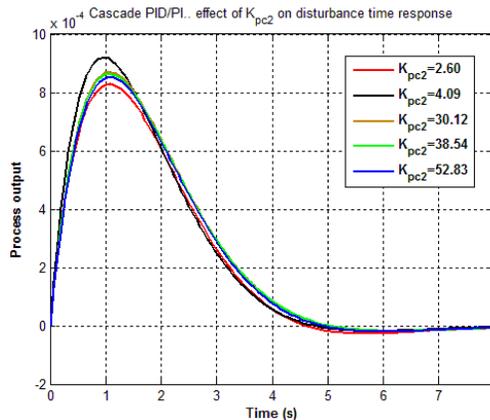


Fig.5 Effect of  $K_{pc2}$  on the disturbance time response of the control system.

- (9) The effect of the last controller gain parameter  $K_{i2}$  on the performance of the control system is shown in Fig.6 for a gain value in the range:  $9.92 \leq K_{i2} \leq 49.12$  for the first gain parameter set at a value of 800, the derivative gain  $K_{d1}$  set at 503 and the proportional gain  $K_{pc2}$  set at 2.6. The effect of  $K_{i2}$  is minor specially outside the time of maximum time response to the unit disturbance input.

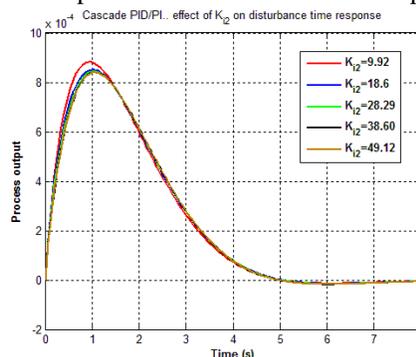


Fig.6 Effect of  $K_{i2}$  on the disturbance time response of the control system.

## VI. COMPARISON WITH OTHER CONTROLLER

- (1) The PID/PI series cascade controller is the third type of cascade controllers used in this series of research papers. The other two cascade controllers investigated are the PI/P cascade controller [14] and the PI/PID cascade controller [13]. The comparison between the effect of using the three types of cascade controllers is shown graphically in Fig.7. The superiority of the present PID/PI controller over the other two types is clear.

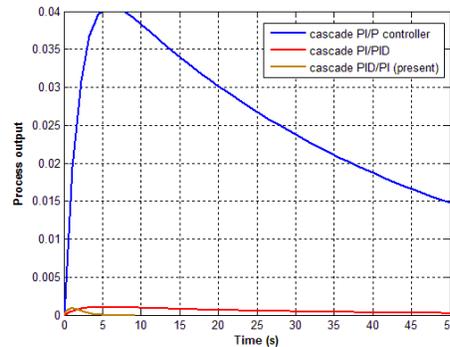


Fig.7 Comparison between time responses using PI/P, PI/PID and PID/PI controllers.

- (2) To investigate the efficiency of the PID/PI cascade controller in rejecting the disturbance associated with the highly oscillating process, its use is compared with more controllers used in this series of research papers. Fig.8 compares the time response of the closed loop control system using the PID/PI control with that using PD-PI controller [20], PI-PD controller [21], IPD controller [22], 2DOF controller [23], PPI controller [24] and PIP controller [25].

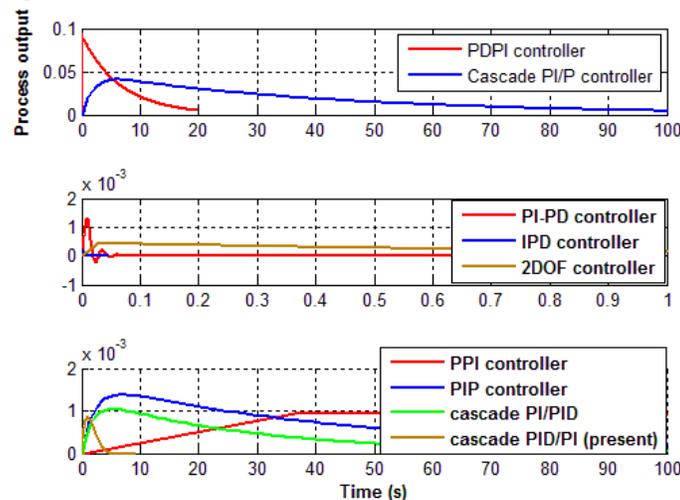


Fig.8 Comparison between time responses using PID/PI and other controllers.

## VII. CONCLUSIONS

- A PID/PI series cascade controller was investigated for disturbance rejection associated with highly oscillating processes.
- MATLAB control and optimization toolboxes were used to tune the controller.
- Five objective functions (ITAT, ISE, IAE, ITSE and ISTSE) were used in the tuning process.
- The ISTSE gave the best tuning results and control system performance during disturbance rejection.
- The effect of the controller five parameters on the disturbance time response of the closed-loop control system was investigated.
- The integral gain of the PID sub-controller had no effect on the control system dynamics.
- The proportional and derivative gain of the PID sub-controller had remarkable effect on the control system dynamics.
- With the PID/PI cascade controller it was possible to go down to a maximum disturbance response of 0.00084 at a time of only 1.08 second.



- The PID/PI cascade controller was compared with the other PI/P and PI/PID cascade controllers. It was superior compared with them.
- The PID/PI cascade controller was also compared with PD-PI, PI-PD, IPD, 2DOF, PPI and PIP controllers for the same objectives. It could compete with all of them except the IPD controller.

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